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OXYGEN RELATIONSHIPS IN STREAMS. METHODOLOGY TO BE APPLIED WHEN DETERMINING THE CAPACITY OF A STREAM TO ASSIMILATE ORGANIC WASTES

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OXYGEN RELATIONSHIPS IN STREAMS

Methodology to be Applied When Determining The Capacity of a Stream to Assimilate Organic Wastes

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PURPOSE

This manual has been compiled to facilitate water resources studies by providing a readily available source of information covering methodology and some of the concepts used to estimate oxygen relationships in a stream. There are many fine technical writings, journals, and books covering oxygen relationships in a stream; however, a need has developed to have some of the methodology and analysis techniques available in a reference manual. It is hoped this manual can fill the need.

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Eckenfelder & O'Connor - "Biological Waste and Treatment," Pergamon Press, 1961

Fair & Geyer - "Water Supply and Waste-Water Disposal," Wiley & Sons, Inc., 1954

E. C. Tsivoglou - "Natural Self-Purification of a Polluted Stream," (RATSEC Training Course Manual), Public Health Service

Phelps - "Stream Sanitation," Wiley & Sons, Inc., 1944

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I. Introduction

The availability of water of suitable quality has determined the location of most past and present civilizations. Therefore, it must be concluded that future population and economic growth will rely heavily upon the availability of water.

To insure maximum utilization of water resources, many variables must be considered, as maximum utilization requires water quality to be suitable for multi-purpose uses. Multi-purpose uses include water supply, recreation, support of fish and aquatic life, plus waste assimilation and transport. To maintain water quality suitable for these beneficial uses requires development of specific water quality objectives; and these objectives or criteria must be basically concerned with the introduction, elimination, or control of waste materials in the water.

A principal problem in the analysis or evaluation of water resources is the coincidence of adverse natural stream conditions with periods of maximum water use. Normally, summer and early fall months are the most critical periods for water quality and quantity, because of high temperatures, low flows, and maximum water use. Prior to the formulation of specific water management programs, it is necessary to have some knowledge of the physical, chemical, and biological characteristics of available water resources, especially surface waters.

This presentation will set forth some of the available methods for analyzing and evaluating effects of biologically degradable organic wastes on stream quality. The indicator of water quality used throughout this manual is the dissolved oxygen content (D. O.), while the measure used for expressing organic pollution in the following discussions and calculations is the bio-chemical oxygen demand (BOD). In presenting the following methodology, it is assumed that there are multiple stream uses, including waste assimilation and transport, and that a minimum quality objective is to be maintained.

II. Dissolved Oxygen Relationships In A Stream

Aerobic stabilization of organic matter within a stream environment results in utilization of oxygen dissolved in the water. Oxygen is generally available or becomes available from the following five sources:

1. Dissolved oxygen within the stream,
2. Oxygen transfer to the water from the atmosphere (stream aeration or reaeration),

3. Dissolved oxygen within the waste,
4. Oxygen produced by stream flora (algae),
5. Chemical reduction of compounds (e.g., nitrites and nitrates, $2NO_3 \rightleftharpoons 2NO_2 + O_2$

Where relatively high nutrient concentrations, long periods of sunlight, and low turbidity exist, stream flora may cause considerable fluctuations in dissolved oxygen and BOD levels. When significant algal populations exist, oxygen concentrations usually exhibit diurnal fluctuations; higher D. O. concentrations during periods of sunlight and lower concentrations at night. Stream flora utilize oxygen continuously in respiration; however, at night, or on overcast days, a lack of sunlight may cause the respiration rate (rate of oxygen use) to exceed the rate of oxygen production from photosynthesis. When this condition occurs, the flora can exert a considerable oxygen demand causing the large diurnal fluctuations. Upon death, algae can be responsible for a major BOD loading in the stream. Although nutrient concentrations are becoming major problems in more and more areas, the effects of nutrient balance and resulting flora on a stream's oxygen balance are considered beyond the scope of this presentation; therefore, have been omitted.

The measurement of the D. O. concentration in a laboratory sample is a reasonably accurate determination; however, it must be recognized that a stream is a dynamic system. It is unrealistic to consider a single D. O. measurement as representative of a stream under all conditions, because of variations in flow, temperature, waste load, and other independent variables.

The quantity of oxygen required to stabilize a given amount of oxygen material is relatively constant. Approximately the same number of pounds of oxygen are required to stabilize an organic waste at 30^{C} as would be required at $5^{o}C$. However, because of the effect of temperature on biological metabolic rates, the time required for stabilization of a waste is greatly increased when temperatures are lowered.

Oxygen, as it occurs in a stream, is measured as a concentration, milligrams per liter (mg/l). To convert concentration to mass flow rate (pounds/day) requires the following calculation:

lbs oxygen/day = mg/l D. O.) x (MGD stream flow) x (8.34 lbs/gal)

or since 1 MGD = 1.55 cfs

lbs oxygen/day = (mg/l) x (cfs stream flow) x 5.38 lbs/cfs)

This calculation gives the total mass flow rate of oxygen dissolved in the stream, for a particular location and time of sampling. When a D. O. objective has been established for the

stream, the dissolved oxygen in the stream available for oxidation of organic materials would be:

O_2 available (mg/l) = present stream D. O. (mg/l) - stream D. O. objective (mg/l)

This calculation shows that in many cases only a portion of the total D. O. in a stream may be utilized for oxidation, as the stream D. O. objective (minimum allowable D. O.) may represent the greater portion of the stream D. O.

Maximum dissolved oxygen concentrations in water (saturation D. O's.) are dependent upon temperature, as illustrated by Figure 1. Engineering calculations to determine stream waste assimilation capacities[1/] are dependent upon the amount of oxygen available and the rate of oxygen utilization. Both of these variables are temperature dependent. Figure 1 gives oxygen solubility in water in degrees Centigrade and degrees Fahrenheit for the usual range of temperatures. Since less oxygen may be dissolved in a stream at warm temperatures, the amount of D. O. available for oxidation of wastes is also decreased. When periods of warm stream temperatures and low stream discharges coincide, primarily during the summer and fall, the critical conditions for waste assimilative capacity occur.

III. Biochemical Demand (BOD)

BOD is a laboratory measurement of the amount of oxygen that is required or utilized in the stabilization of an organic waste. A graphical representation of a typical curve of BOD exertion vs time is shown in Figure 2. Figure 2 shows that there are two stages of BOD exertion; the first (carbonaceous) stage is satisfied in approximately 20 days; then the second (nitrogenous) stage exerts its oxygen demand. In most engineering calculations, the carbonaceous BOD is the measurement used; therefore, all references to BOD in this report refer to the carbonaceous BOD. It is significant to note that BOD results may show considerable fluctuations, primarily because the BOD test is based upon the functioning of a biological system which tends to be quite variable. Averaging values over a period of time increases reliability of the results; however, even the averaged values should not be considered absolute, regardless of the type of sampling and/or laboratory technique employed.

1/ Assimilative Capacity - the ability of a system to absorb a change (waste) without destroying the prior uses of the system.

BOD measurements to determine oxygen utilization are usually carried out over a 5-day period at a temperature of 20°C; hence the reporting terminology "5-day BOD." Complete stabilization of a waste normally requires a much longer period than 20 days; however, the ultimate BOD (oxygen required to completely stabilize the carbonaceous BOD) can be calculated from the 5-day value. The deoxygenation of wastes (rate of BOD exertion) through the first stage reaction is generally approximated by a first order equation. The rate of reaction is directly proportioned to the amount of organic material present. This reaction is represented mathematically by the differential equation:

$$\frac{dL}{dt} = K_L L$$

K_L = reaction rate constant

L = amount of BOD remaining

$\frac{dL}{dt}$ = rate of BOD utilization

On integration this becomes:

$L = L_a e^{-K_1 t}$ and represents first stage curve, Figure 2.

L = BOD remaining

L_a = ultimate BOD of the waste

K_1 = deoxygenation rate constant (Base "e")[1/]

t = elapsed time

Or: $y = L_a - L_a e^{-K_1 t} = L_a (1 - e^{-K_1 t})$

y = amount of BOD utilized

L_a = ultimate BOD of the waste

K_1 = deoxygenation rate constant (Base "e")

t = elapsed time

1/ Convention indicates K_1 be used for Base "e" notation and k_1 be used for Base "10" notation.

Reaction rates in the above equations are expressed in terms of Base "e;" however, they may also be expressed to the Base "10." Considerable care must be exercised to insure the proper base is being used. Base "e" figures may be converted to Base "10" figures in the following manner:

$$K_1 \text{ (Base "e")} = k_1 \text{ (Base "10")} (2.3)$$

To calculate the ultimate BOD value for a waste from the 5-day BOD, the following relationships are used:

$$\text{Ultimate BOD } (L_a) = \frac{y \text{ (5-day)}}{1 - e^{-K_1(5)}} = \frac{\text{5-day BOD (100)}}{\% \text{ BOD satisfied in 5 days}}$$

The per cent BOD satisfied in 5 days is dependent upon temperature and the bio-degradability of the waste. Figure 3 indicates the proportion of BOD removed in a given time period with a specified reaction rate constant. When laboratory data on a given waste or stream are not available, 5-day BOD removal is usually taken as 68 - 83 per cent which, from Figure 3, indicates K_1 to be 2.3 - 3.45. The above discussion was centered on 5-day BOD values, but any time period can be taken to use in approximating the ultimate BOD; however, the 5-day values give more consistent results than the BOD values for shorter time intervals.

In addition to providing more consistent values for the ultimate BOD, the 5-day BOD is utilized to reduce the time lag between the sample collection and laboratory results; to reduce incubator space requirements in the laboratory; and to allow comparison of results by the selection of standard conditions. These factors are extremely important when volume sampling is being done.

IV. Deoxygenation Rate Constant (K_1 or k_1)

The deoxygenation rate constant is generally defined as the rate of oxygen utilization by a waste material, within a particular environment. Characteristics which influence the rate constant are as follows:1/

a. Turbulence - The speed of many chemical reactions is increased by turbulence. The same effect is

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1/ Eckenfelder and O'Connor, "Biological Waste Treatment," Pergamon Press, 1961.

expected on biochemical reactions because of the greater opportunity that is afforded for contact between the organic matter and the biological population.

b. Biological growth on the stream bed - A biological population attached to the stream bed, particularly in shallow, turbulent waters containing rocks, will greatly increase the reaction rate. This biological growth would be similar to that found in a trickling filter at a waste treatment facility.

c. Immediate chemical demand - Some wastes contain chemical substances or end products of anaerobic decomposition which exert an immediate chemical demand. Upon reaching a stream, these wastes have an almost instantaneous uptake of D. O. and reach stability by chemical rather than biological processes.

NOTE: Significant amounts of immediate chemical demand can be separated out when determining the biological reaction rate constant. This demand is measured as the BOD exerted in the first few minutes; a period too short to allow biological activity to become significant, and is shown when plotting BOD exerted vs. time.

d. Nutrients - Nutrients (nitrogen, phosphorus, and perhaps other substances) are necessary for the support of stream flora and fauna (plant and animal life). Lack of nutrients (fertilizer) restrains the growth of biological organisms responsible for the oxidation processes. If a waste of the receiving stream is deficient in nutrients, oxidation will proceed at a much slower rate than in a laboratory when a standard BOD dilution water is used.

e. Lag in oxidation processes - If an insufficient biological population is present in a stream and/or waste, some interval of time will pass before normal oxidation occurs. The lag is the time required to build up an adequate population.

f. Dilution or waste concentration - Large volumes of water dilute waste discharges which, in turn, reduce the opportunity for contact between the organic material and the biological population. If waste

concentration is very low, the reaction rate will be considerably reduced.

Other factors which must be considered when trying to establish the deoxygenation rate in a stream include flocculation and sedimentation, scour, and volatilization. Although these factors do not affect the actual reaction rate, they can result in the misinterpretation of field measurements. Flocculation and sedimentation, as well as volatilization, give results which indicate marked decreases in BOD. This would indicate that the rate of removal of organic material in the stream is much higher than actually exists. However, organic sediment will exert its BOD at a later date. Scour can indicate a low reaction rate, because sediments within the area may be brought into suspension, indicating little BOD exertion, although considerable oxidation may have taken place.

The deoxygenation rate varies significantly and directly with temperature. The variation is a result of increased biological activity at higher temperatures, and limited or reduced activity at lower temperatures. The generally accepted relationship between the reaction rate constant and temperature is as follows:

$$K_1 \text{ @ any temperature } T = K_1 \text{ @ } 20^{\circ}C \times 1.047^{(T-20)}$$

Where T = stream temperature in $^{\circ}C$

Figure 4 shows numerical values of the factor, $1.047^{(T-20)}$, plotted over the normal range of temperatures. The base temperature of $20^{\circ}C$ is used since this agrees with the laboratory incubation temperature at which the reaction rates are normally evaluated.

Expected values for the deoxygenation rate constant are given in the table below:

TABLE I

Stream Characteristics	K_1 (base e)	k_1 (base 10)
Heavily polluted streams	0.23 - 0.69 generally 0.41	0.1 - 0.3 generally 0.17
Streams not subject to immediate pollution	0.19 - 0.36	0.08 - 0.15
Streams with small pollution loads	0.12 - 0.23	0.05 - 0.10
Sludge deposits (does not consider waste loads in the stream above the sludge deposit)	0.023 - 0.19	0.01 - 0.08

V. Determination of Deoxygenation Rate Constant (K_1)

Given BOD-time series data from laboratory analyses, there are several methods of obtaining the reaction rate of the incubated sample. Each method gives approximately the same result; therefore, the simplest method has been selected for presentation here. It must be realized that reaction rates calculated from laboratory observations are not necessarily the same as those in the stream. The biological environment of a dynamic system, such as a stream, cannot be duplicated in the laboratory.

Example I

Daily Difference Method

The following results were obtained from a short-term analysis of a sample by determining the BOD exerted each day for a period of 7 days. It is not necessary to make the BOD determinations at the time intervals indicated, but the time of observation must be carefully noted and the data plotted accordingly.

Step 1. Laboratory Results

t (days)	y (BOD exerted) mg/l
0	0
1	0.7
2	1.4
3	1.6
4	1.7
5	1.9
6	2.2
7	2.3

Step 2. Graphic Plot

Plot the laboratory data on arithmetic graph paper as shown in Figure 5a.

Step 3. Curve Fitting

Fit a smooth curve through the plotted points (Figure 5a). From this smooth curve, take BOD exerted values (y) for the corresponding times (t). This procedure adjusts or averages the BOD readings.

t (days)	Adjusted Values y (BOD exerted)
0	0
1	0.72
2	1.25
3	1.55
4	1.80
5	2.00
6	2.20
7	2.35

Step 4. Daily Differences

Using the adjusted BOD values, calculate the differences over equal increments of time. A daily period is generally satisfactory and most frequently used.

Time Interval (days)	BOD Difference (mg/l)	Plotting Position (used in Step 5)
0-1	0.72	1/2
1-2	0.53	1 1/2
2-3	0.30	2 1/2
3-4	0.25	3 1/2
4-5	0.20	4 1/2
5-6	0.20	5 1/2
6-7	0.15	6 1/2

Step 5.

Plot the incremental data on semi-log paper, with time on the linear scale and the BOD differences on the log scale, as shown in Figure 5b. Values are usually plotted at the mid-point of the time interval.

Step 6.

Draw a line of best fit through the points, extending the line to time zero. The plotted values may not exhibit a straight line relationship beyond $t = 4$, or $t = 5$, indicating nitrification is beginning.

Step 7.

Using the incremental values of BOD at selected times to get the maximum time interval having good correlation with the curve, in this case $t = 0$ and $t = 4$ to eliminate the portion affected by nitrification, make the following calculation:

$$\frac{\text{BOD/day @ } t = 4}{\text{BOD/day @ } t = 0} = \frac{0.195}{0.75} = 0.26$$

This indicates that after 4 days, 26% of the BOD remains to be oxidized, and that 74% of the ultimate (First Stage) BOD has been stabilized.

Step 8.

From Figure 3, using t = 4, and y = 26%, the laboratory K_1 is determined to be approximately 0.32 @ 20°C (the laboratory incubation temperature.

Figure 3 is an expedient over using the following equations to calculate the deoxygenation rate constant.

a. $K_1 = 1/t \ln \frac{\Delta yo}{\Delta yt}$ NOTE: Δy figures are ordinates of Figure 2b.

b. $K_1 = \frac{4.6 - \ln(\frac{\Delta yt}{\Delta yo}100)}{t \text{ or } \Delta t}$ NOTE: Δy figures are ordinates of Figure 2b.

Step 9.

The ultimate BOD of this sample may be calculated as follows:

@ t = 4, 74% of the BOD has been oxidized; from Figure 2a, the BOD exerted @ t = 4 is 1.6 mg/l.

Therefore:

74% of the ultimate BOD (L_a) = 1.6 mg/l

$0.72 L_a = 1.6$ mg/l

$L_a = \frac{1.6}{0.74} = 2.16$ mg/l

The ultimate BOD can also be calculated from the following relationships:

a. $L_a = \frac{yt}{1 - e^{-K1t}}$

yt = BOD exerted after time t (from Figure 2a)

K_1 = deoxygenation rate constant calculated above

t = elapsed time

b. $L_a = yt \frac{\Delta yo}{\Delta yo - \Delta yt}$

yt = BOD exerted after time "t" (from Figure 5a)

Δyo = ordinate @ time zero, Figure 5b.

Δyt = ordinate @ time "t", Figure 5b.

VI. Determination of Stream K_1 Rate

A combination of field measurements and the laboratory (bottle) K_1 rate is used to calculate the deoxygenation rate of a stream reach. This determination requires stream BOD values for at least two sampling stations downstream from a waste discharge. The stations should be a reasonable time of flow apart, and there must not be another source of waste entering the reach. The magnitude of the waste is determined by locating a sampling point above as well as below the point of introduction of the waste. Any tributaries entering between the sampling points must be considered; however, sampling points should be selected to avoid this situation when at all possible.

Derivation

1. It has previously been shown that the BOD-time curve is logarithmic (Figure 2). The curve is expressed mathematically as follows:

$L_t = L_a (e^{-K_1 t})$ or $L_t = L_a (10^{-K_1 t})$

L_t = BOD (mg/l) remaining after time "t"

K_1 = deoxygenation rate, Base e

t = time interval (days)

L_a = ultimate first

2. By taking the ln (log) of both sides of the equation, it is possible to obtain a straight line relationship simplifying the calculation of K_1.

$L_t = L_a (e^{-K_1 t})$

$\ln L_t = \ln L_a + (-K_1 t$

$\ln \left(\frac{L_t}{L_a}\right) = -K_1 t$

3. Assuming data is available for two sampling stations, B and C, with flow time "t" between stations; ln L_{a_B} is the natural logarithm of the ultimate BOD at point "B". Likewise, ln L_{a_C} is the natural logarithm of the ultimate BOD at point "C."

Therefore, given time zero at point B:

$$L_a = L_{a_B}$$

$$L_t = L_{a_C}$$

$$-K_1 t = \ln \left(\frac{L_t}{L_a}\right) \quad \text{(from Step 2)}$$

$$K_1 = \frac{-1}{t} \ln \left(\frac{L_{a_C}}{L_{a_B}}\right) = \frac{1}{t} \ln \left(\frac{L_{a_C}}{L_{a_B}}\right)$$

4. Using the ultimate BOD values at points A and B, plus the time of flow between the points, K_1 can be estimated a direct calculation.

A graphical solution for the determination of the stream K_1 utilizes the same equation; however, the graph itself may indicate many characteristics of the stream, waste, and environment. The graphical approach is of greatest value when points of significant waste discharge are known or suspected. The graph provides a method for estimating waste strengths, reaches where sedimentation may be occurring, and the deoxygenation rate. This technique is usually referred to as the Oxygen Balance Calculation.

Procedure for Constructing Figure 6:

Step 1.

Using K_1 values computed from long-term BOD's in the laboratory, adjust the stream 5-day BOD's (as determined in the laboratory at 20°C) to ultimate BOD's.

Step 2.

Determine stream flows for the days of sampling.

Step 3.

Convert ultimate BOD's from mg/l to pounds BOD/day in the stream.

Step 4.

Plot a BOD profile of the stream. The most useful plot is log BOD pounds/day vs time of flow; however, this is only practical when the flow rate is approximately uniform over the stream reach during the sampling period. With variable flows, a plot of log BOD vs stream mile is more practical. Figure 6 shows log BOD's plotted against stream miles and points A, D, E, H, I, L, and M represent measured values of stream BOD. Points B', C', F', G', J', and K', represent values or points located by extrapolation.

Step 5.

Beginning with the first two plotted points downstream from a waste discharge, use the following procedure:

a. Draw a straight line between the first two points below the waste discharge, points D and E. For more accurate results, there should not be any significant waste or flow contribution between points D and E.

b. Project the line fixed by points D and E to the point of waste discharge, point C' and downstream to point F'.

c. Draw a line parallel to C'-D-E-F', through point A and extending the line to point B'.

d. The number of pounds of BOD represented by the distance between points C' and B' indicates the amount of waste contribution between points A and D, probably contributed at point #1.

e. Continue for other points downstream.

The following conclusions may be reached after an analysis of Figure 6:

1. Slope of line D-E indicates the rate of BOD removal for the reach of stream bounded by points C' and F'.

2. The interval B'-C' indicates the amount of waste contributed at point #1.

3. The interval F'-G' indicates the waste contributed at point #2. Slope of line H-I indicates the BOD

removal rate for the waste concentration below point #2. The lesser slope indicates that the new waste may be more difficult to oxidize biologically or contains toxic substances which inhibit biological activity.

4. The steep slope of line L-M indicates the waste mixture below point #3 has a very rapid reduction of BOD. The probable reasons for the rapid reduction would include:

   a. The waste mixture below point #3 is easily bio-degradable.

   b. There is a chemical oxygen demand.

   c. Sedimentation is occurring.

   d. Shallow, rocky stream having heavy biological growth on the rocks which perform like a trickling filter.

VII. Reaeration

Reaeration is the process of oxygen transfer from air into water to bring the partial pressure of oxygen in the stream into equilibrium with the partial pressure of oxygen in the atmosphere. Absorption of oxygen in a stream occurs only at the surface layer of water. The water surface is readily saturated; however, complete oxygenation throughout the depth of the stream depends upon turbulence to mix the saturated surface layer with the lower levels and expose a new surface layer to the atmosphere. Based on the above, a swift, shallow, turbulent stream will have a much higher oxygen uptake rate than a deep stream with laminar flow and uniform channel characteristics.

Reaeration becomes operative when the oxygen content of water is below the saturation level. Reaeration proceeds as a first order reaction, as shown in Figure 7. The mass transfer rate for oxygen is proportional to the reaeration rate constant (K_2) and the oxygen deficit (D).

$r = K_2D$

r = mass transfer rate for oxygen

K_2 = reaeration rate constant

D = oxygen deficit = (saturation D. O. - stream D. O.)

Thus, higher oxygen deficits produce higher rates of oxygen transfer (r) for a given reaeration rate constant. Temperature also plays an important role in reaeration, since the dissolved oxygen saturation values and the rate of oxygen transfer (K_2) are temperature dependent.

VIII. Reaeration Coefficient - K_2 (base e) or k_2 (base 10)

The reaeration coefficient is defined as the rate constant controlling the process of reoxygenation, or rate of oxygen absorption in a stream under given physical conditions. Factors which affect K_2 values include: stream depth, velocity, turbulence (physical characteristics of the stream bed), temperature, and dissolved solids such as chlorides. Values of K_2 (base e) usually range between 0.1 and 12.0 per day; the lower values representative of deep, slow-moving rivers; and the higher values for shallor and/or rapid streams.

After a stream K_2 value has been determined for a particular temperature condition, the value can, through the use of equations, be computed for any temperature condition. The two most common empirical temperature equations are as follows:

1. $K_{2_T} = K_{2_{20}} \left[1.047^{(T-20)}\right]$

This is identical to the temperature correction for K_1 shown in Figure 4.

2. $K_{2_T} = K_{2_{20}} \left[1.017^{(T-20)}\right]$

Use Figure 8.

The amount of oxygen dissolved in a stream is readily determined by laboratory analysis of stream samples; however, the amount of dissolved oxygen added through reaeration must be calculated. Several methods are available for calculating the reaeration coefficient, but only two will be shown here. The first method utilizes empirical relationships, such as those suggested by O'Connor, Velz, Churchill, and others; while the second method utilizes field observations to set up a "trial and error" solution of the oxygen sag equation.

O'Connor Method

$$K_2 \text{ (base e)} = \frac{(DV)^{1/2}}{H^{3/2}} \times 24 \text{ hr/day}$$

Where K_2 = reaeration coefficient (day^{-1})

v = stream velocity (ft/hr)

H = average stream depth (ft)

D = molecular diffusivity of oxygen in water (0.81×15^{-4} ft^2/hr @ 20°C)

Trial and Error Solution for K_2 or (k_2)

1. The field measurements of D. O., BOD, temperature, and time of flow.

2. Compute K_1 as discussed in Section VI.

3. Compute D. O. deficits at points A and B.

4. Assume values of K_2 and solve the oxygen sag equation for D_B.

$$D_B = \frac{K_1 L_a}{K_2 - K_1} \left[e^{-K_1 t} - 10^{-K_2 t} \right] + D_A \left(10^{-K_2 t}\right)$$

Where D_B = oxygen deficit at point B

K_1 = deoxygenation coefficient

K_2 = reaeration coefficient

L_a = ultimate BOD @ point A

t = time of flow between points A and B

D_A = oxygen deficit at point A.

5. Plot the calculated values of D_B vs the assumed K_2 values.

6. Draw a smooth curve through the plotted points.

7. Using the actual (measured or observed) D_B, read the corresponding K_2 value from the curve.

The "trial and error" solution seldom produces good results, but should be considered a reference or check calculation for empirical solutions.

IX. Oxygen Sag Equation

Introduction of an organic waste into a stream causes deoxygenation, then reaeration to be effected simultaneously. The resultant dissolved oxygen concentrations plotted vs time or stream mile define the "dissolved oxygen sag." A plot of the D. O. profile of a stream (D. O vs time of flow) below a waste discharge gives a spoon shaped curve for which the mathematical relationships have been developed by Streeter and Phelps. Modification of these relationships have been proposed by Velz, Thomas, and many others.

When using the oxygen sag equation, it must be recognized that complex biological processes, such as waste assimilation in a stream, are extremely difficult to express mathematically. Therefore, the sag equation can only approximate actual stream conditions and limitations of the oxygen sag relationship must be recognized and appreciated prior to any analysis.

The sag equation utilizes oxygen deficits to express stream D. O. conditions. The equation is expressed as follows:

Base "e"

$$D_t = \frac{L_a K_1}{K_2 - K_1} \left[e^{-K_1 t} - e^{-K_2 t} \right] + D_a e^{-K_2 t}$$

Where D_t = D. O. deficit (mg/l below saturation) after time "t"

L_a = ultimate BOD within the stream at point A (mg/l)

K_1 = deoxygenation coefficient ($days^{-1}$)

K_2 = reaeration coefficient ($days^{-1}$)

t = elapsed time in days between point A and any downstream point, point B

D_a = initial D. O. deficit at point A (mg/l)

The equation may also be expressed in terms of Base "10"

$$D_t = \frac{L_a k_1}{k_2 - k_1} \left[10^{-k_1 t} - 10^{-k_2 t} \right] - D_a 10^{-k_2 t}$$

In engineering practice, the maximum D. O. deficit (lowest D. O. value, or worst D. O. condition to occur) is of major importance. A typical oxygen sag is shown in Figure 9 which illustrates the

various terms involved. The critical time and critical deficit indicated in Figure 9 give the relative location and magnitude of the worst condition to be expected in the stream. These values are used to determine the maximum waste load that can be discharged to a stream and yet maintain beneficial water uses; aid in the determination of allowable uses; and provide the basis for programing waste and/or dilution water discharges to meet the established dissolved oxygen quality objective.

The time to the critical point (t_c) is determined by differentiating the oxygen sag equation and setting the first derivative equal to zero.

$$\frac{dD}{dt} = \frac{K_1 L_a}{K_2 - K_1} \left[-K_1 e^{-K_1 t} + K_2 e^{-K_2 t} \right] - K_2 D_a e^{-K_2 t} = 0$$

The above equation simplifies to the following relationship defining t_c:

$$t_c = \frac{1}{K_2 - K_1} \log_e \frac{K_2}{K_1} \left[1 - D_a \frac{(K_2 - K_1)}{L_a K_1} \right]$$

The basic differential equation expressing these processes is:

$$\frac{dD}{dt} = K_1 L - K_2 D$$

At the sag point, the deoxygenation rate equals the rate of oxygen transfer into the stream, giving rise to the following relationships:

$$\frac{dD}{dt} = 0 = K_1 L - K_2 D$$

$$K_1 L_c = K_2 D_c$$

where D_c = critical deficit = D_c

L_c = amount of ultimate BOD remaining after the critical time t_c

since $L_c = L_a e^{-K_1 t_c}$

then $K_2 D_c = K_1 L_a e^{-K_1 t_c}$

$$D_c = \frac{K_1}{K_2} L_a e^{-K_1 t_c}$$

When using the oxygen sag equation, it is convenient to let $\frac{K_2}{K_1}$ be represented by a factor "f" ($f = \frac{K_2}{K_1}$). The parameter "f"

occurs frequently and is known as the stream purification factor, providing a means of defining the combined characteristics of stream and waste. A stream with $K_1 = 0.3$ and $K_2 = 1.2$ has an "f" value of 4, as does a stream with $K_1 = 1.0$ and $K_2 = 4.0$. Higher "f" values indicate a greater capability of a stream to assimilate organic wastes.

Temperature variations may or may not affect the value of "f." If the temperature correction factor $(1.047)^{(T-20^oC)}$ is used for both K_1 and K_2 values, temperature changes do not alter the purification factor. If K_2 is considered to vary by $(1.017)^{(T-20^oC)}$, temperature changes must be considered. Many sanitary engineers feel the refinement offered by the $(1.017)^{T-20}$ factor is not warranted and that the $(1.047)^{T-20}$ factor gives more reliable results.

Typical values of the self-purification factor have been given by Fair and Geyer in their text. Typical values of "f" for various stream conditions are shown in Table II below:

TABLE II - TYPICAL VALUES OF THE SELF-PURIFICATION FACTOR "f"

Nature of Receiving Waters	"f" Value @ 20°C.
Small ponds and backwaters	0.5 - 1.0
Sluggish streams, large lakes or impoundments	1.0 - 1.5
Large streams of low velocity	1.5 - 2.0
Large streams of moderate velocity	2.0 - 3.0
Swift streams	3.0 - 5.0
Rapids, waterfalls, etc.	5.0 and up

X. Uses of the Oxygen Sag Equation

The principal use of the sag equation is calculation of maximum allowable organic loads that may be imposed upon a stream and still maintain specified stream D. O. objectives. This technique allows optimum location of points of waste discharge, to maximize stream utilization and maintain maximum water quality. Three methods are generally available for solving assimilative capacity problems:

1. Manual solution of the sag equation (trial and error not recommended because of time required).

2. Graphical solution (recommended).

   a. Method published in course manual for "Water Quality Studies," Sanitary Engineering Center, Cincinnati, Ohio. (Discussed here)

   b. Method proposed by T. A. Wastler, Public Health Service, in Sewage and Industrial Wastes, September 1958.

3. Computerized solution (large scale and repetitive analyses).

Example II

Graphical Solution of the Assimilative Capacity Problem by Method 2a:

Given:

$K_1 = 0.44$ @ 20^oC

$K_2 = 1.1$ @ 20^oC

$f = \frac{1.1}{0.44} = 2.5$

Stream temperature = 28^oC and $C_s = 7.92$ mg/l

$D_a = 1.0$ mg/l

Minimum acceptable D. O. = 4.0 mg/l

$D_c = C_s - 4.0 = 7.92 - 4.0 = 3.92$ mg/l

Use Figure 10a for the solution, since $D_a = 1.0$ mg/l. Figures 10b and 10c may be used when $D_a = 2.0$ or 3.0 mg/l, respectively. Although field measurements do not give D_a values of exactly 1.0, 2.0, or 3.0 mg/l, an interpolation of values obtained from the tables will provide very close approximations.

Solving for the maximum allowable ultimate BOD concentration, using Figure 10a, a value of about 17 mg/l is obtained for L_a.

Example III

Use of the Graphical Method to Determine Minimum D. O. Concentrations Following the Introduction of a Waste Into a Stream:

Given:

5-day BOD of Waste = 100 mg/l

5-day BOD of Stream = 3 mg/l

Waste Flow = Q_w = 60 mgd = 60 mgd x $\frac{1.55 \text{ cfs}}{\text{mgd}}$ = 93 cfs

Stream Flow = Q_s = 1000 cfs

Temperature of Stream = 20°C and C_s = 9.17 mg/l

D_a = 1.5 mg/l

K_1 @ 20°C = 0.23
K_2 @ 20°C = 0.69

$f = \frac{0.69}{0.23} = 3.0$

5-day BOD of stream and waste mixture =

$$\text{5-day BOD mix} = \frac{Q_s (BOD_s) + Q_w (BOD_w)}{Q_s + Q_w}$$

$$= \frac{1000 \text{ cfs } (3 \text{ mg/l}) + 93 \text{ cfs } (100 \text{ mg/l})}{1000 \text{ cfs} + 93 \text{ cfs}}$$

$$= 11.25 \text{ mg/l}$$

From Figure 3 the 5-day BOD for K_1 = 0.23 represents 0.68 of L_a.

L_a of mixture = $\frac{11.25 \text{ mg/l}}{0.68}$ = 19.4 mg/l

From Figure 10a with D_a = 1.0 mg/l and L_a = 19.4 mg/l

D_c = 4.0 mg/l

From Figure 10b with D_a = 2.0 mg/l and L_a = 19.4 mg/l

D_c = 4.25 mg/l

Therefore, for D_a = 1.5 mg/l, the critical D. O. deficit is approximately:

$$\frac{4.25 \text{ mg/l} + 4.0 \text{ mg/l}}{2} = 4.12 \text{ mg/l}$$

The stream D. O. concentration at the sag point (critical point) equals the saturation D. O. minus the deficit.

$$\text{D. O.}_{\text{stream}} = C_s - D_c = 9.17 \text{ mg/l} - 4.12 \text{ mg/l} = 5.05 \text{ mg/l}$$

XI. Stream Assimilation Capacity

The most useful presentation for any reach of stream allows determination of allowable BOD loadings when temperature and flow conditions vary. This is best achieved using a graphical solution.

Example IV

A Procedure for Developing Assimilative Capacity Curves

Given: $D_a = 1.0$ mg/l

$K_1 = 0.65$

$K_2 = 3.9$

$f = \frac{3.9}{0.65} = 6$ and does not vary with temperature fluctuations

Minimum acceptable D. O. = 5.0 mg/l

1. Assume stream temperature = 30°C

   D. O. saturated = $C_s = 7.63$ mg/l

   $D_c = 7.63$ mg/l - 5.0 mg/l = 2.63 mg/l

   From Figure 10a, $L_a = 21.5$ mg/l

2. Assume stream temperature = 20°C

   $C_s = 9.17$ mg/l

   $D_c = 9.17$ mg/l - 5.0 mg/l = 4.17 mg/l

   From Figure 10a, $L_a = 34.5$ mg/l

3. Assume stream temperature = 10°C

   $C_s = 11.33$ mg/l

   $D_c = 11.33$ mg/l - 5.0 mg/l = 6.33 mg/l

   From Figure 10a, $L_a = 53.0$ mg/l

4. Assume stream temperature = 5°C

   C_s = 12.80 mg/l

   D_c = 12.80 mg/l - 5.0 mg/l = 7.80 mg/l

   From Figure 10a, L_a = 66.0 mg/l

5. Using the values obtained in steps 1 through 4, plot Log L_a versus temperature on semi-log paper, as shown in Figure 11.

6. Select temperatures such as 5°C, 10°C, 25°C, from Figure 11, and obtain the corresponding L_a values. Construct a table to convert BOD from mg/l to pounds BOD by assuming flow values.

7. Plot the values for pounds of BOD from the table on log scale versus temperature, then add constant flow lines, as shown in Figure 12.

The family of curves (Figure 12) developed using the preceding steps can now be used to determine necessary flows for assimilation, or maximum BOD loadings, at any temperature. The curves constructed in Example IV are only applicable where D_a = 1.0 mg/l and the minimum acceptable D. O. is 5.0 mg/l. Other families of curves must be constructed for different D_a values or different D. O. objectives.

When making an analysis of stream assimilative capacities, the reduced quality of the waste water returned must be considered. For example, if 8 mg/l is the stream D. O., stream flow is 150 cfs, water use = 40 cfs, D. O. of water (waste) returned = 2 mg/l; then only a portion of the 150 cfs will be available for waste assimilation, i.e., 150 cfs - 40 cfs $\frac{(8 \text{ mg/l} - 2 \text{ mg/l})}{8 \text{ mg/l}}$ or 120 cfs available.

The above analyses assumed that K_1 and K_2 did not vary with fluctuation in stream flows. When there are large flow fluctuations, K_2 will show significant variations, found to follow the following relationship:

$$K_2 = aQ^b$$

Q equals stream flow.

a and b are constants evaluated from the stream cross-section, Q, relationship.

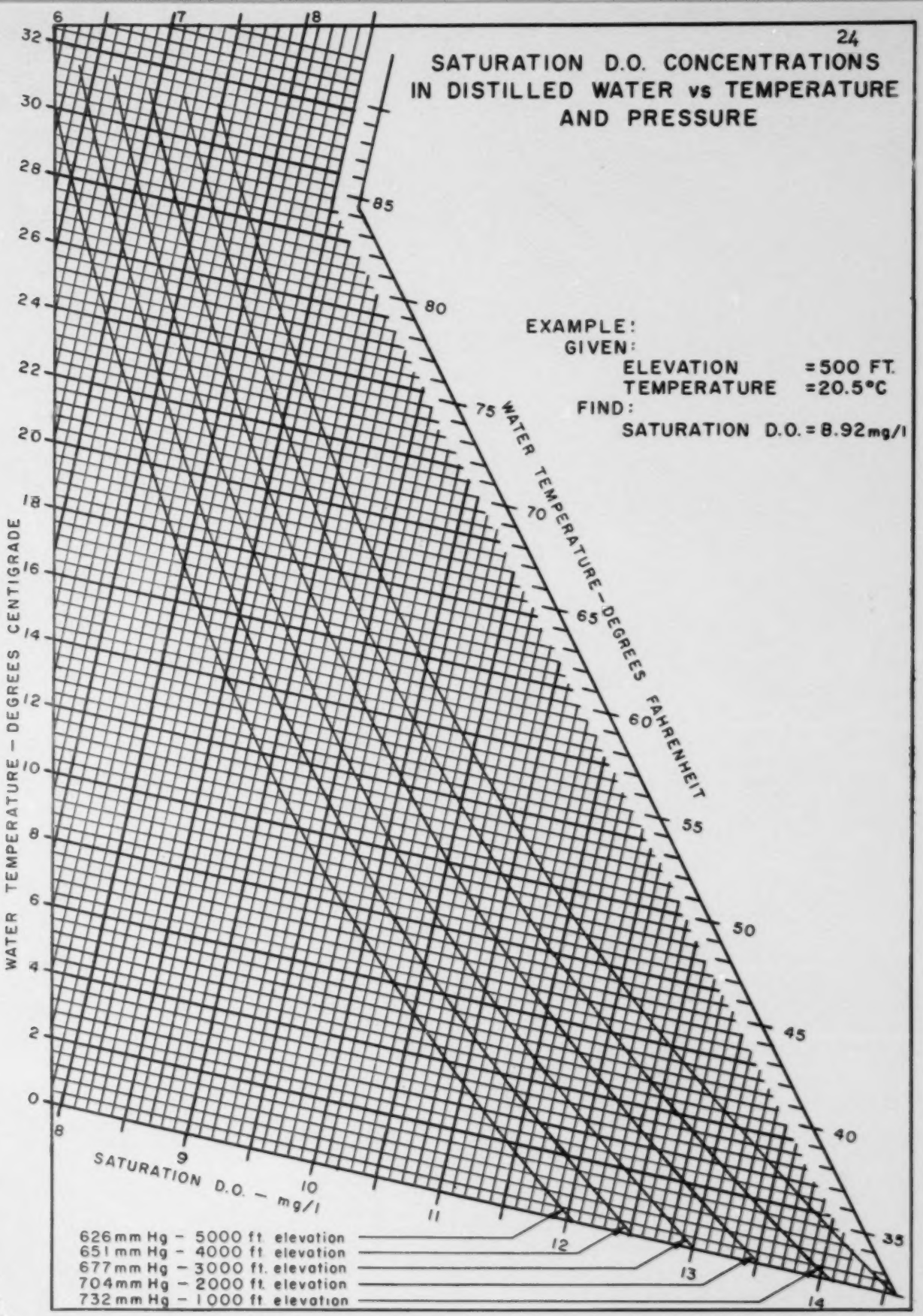
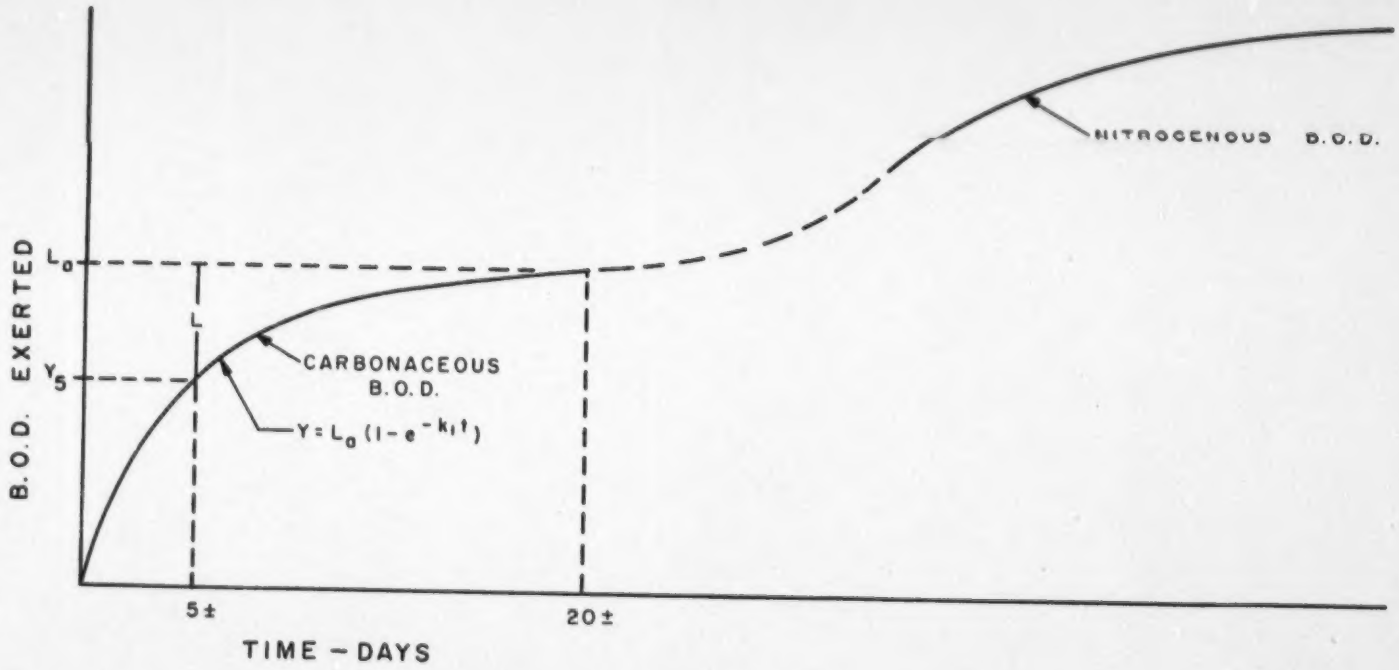

SATURATION D.O. CONCENTRATIONS
IN DISTILLED WATER vs TEMPERATURE
AND PRESSURE
EXAMPLE:
GIVEN:
ELEVATION =500 FT.
TEMPERATURE =20.5°C
FIND:
SATURATION D.O. = 8.92 mg/l
WATER TEMPERATURE - DEGREES CENTIGRADE
WATER TEMPERATURE - DEGREES FAHRENHEIT
SATURATION D.O. - mg/l
626 mm Hg - 5000 ft. elevation
651 mm Hg - 4000 ft. elevation
677 mm Hg - 3000 ft. elevation
704 mm Hg - 2000 ft. elevation
732 mm Hg - 1000 ft. elevation

Figure 1



L_a = ultimate B.O.D.

Y_5 = B.O.D. exerted after 5 days

k_1 = coefficient of de-oxygenation

t = time elasped after introduction of waste to sample, stream, etc.

L = B.O.D. remaining, yet to be exerted

Y = B.O.D. exerted

Figure 2

BOD EXERTION vs TIME

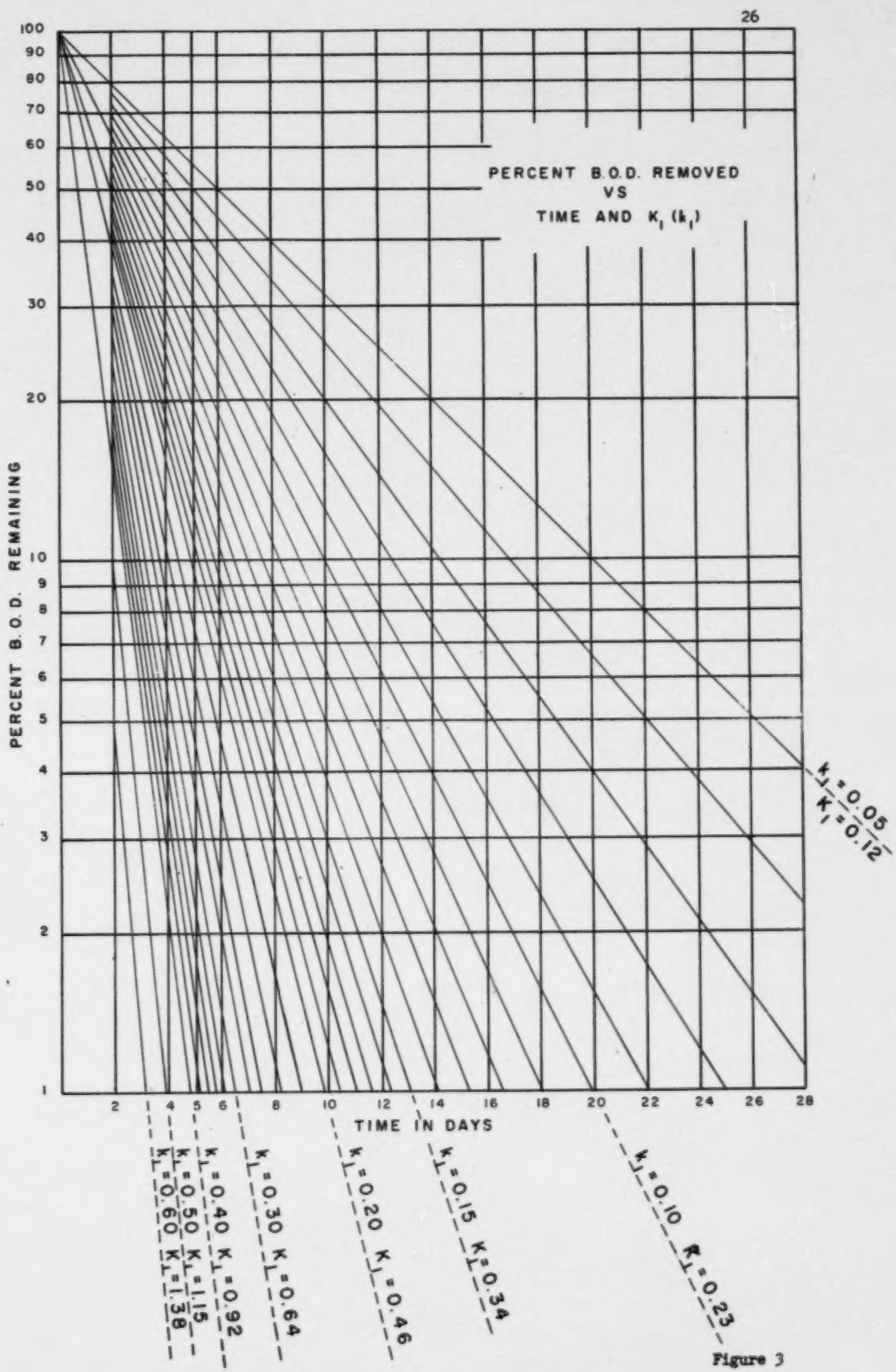

PERCENT B.O.D. REMAINING
100
90
80
70
60
50
40
30
20
10
9
8
7
6
5
4
3
2
1
PERCENT B.O.D. REMOVED
VS
TIME AND K_1 (k_1)
2
4
5
6
8
10
12
14
16
18
20
22
24
26
28
TIME IN DAYS
K_1 = 0.05 / K = 0.12
K_T = 0.60 K_T = 1.38
K_T = 0.50 K_T = 1.15
K_T = 0.40 K_T = 0.92
K_T = 0.30 K_T = 0.64
K_T = 0.20 K_T = 0.46
K_T = 0.15 K_T = 0.34
K_1 = 0.10 K_T = 0.23

Figure 3

EXAMPLE:

K_T (at any T) = K (at 20°) x (FACTOR FROM GRAPH)

K_T (at 25°) = K (at 20°) x (1.26)

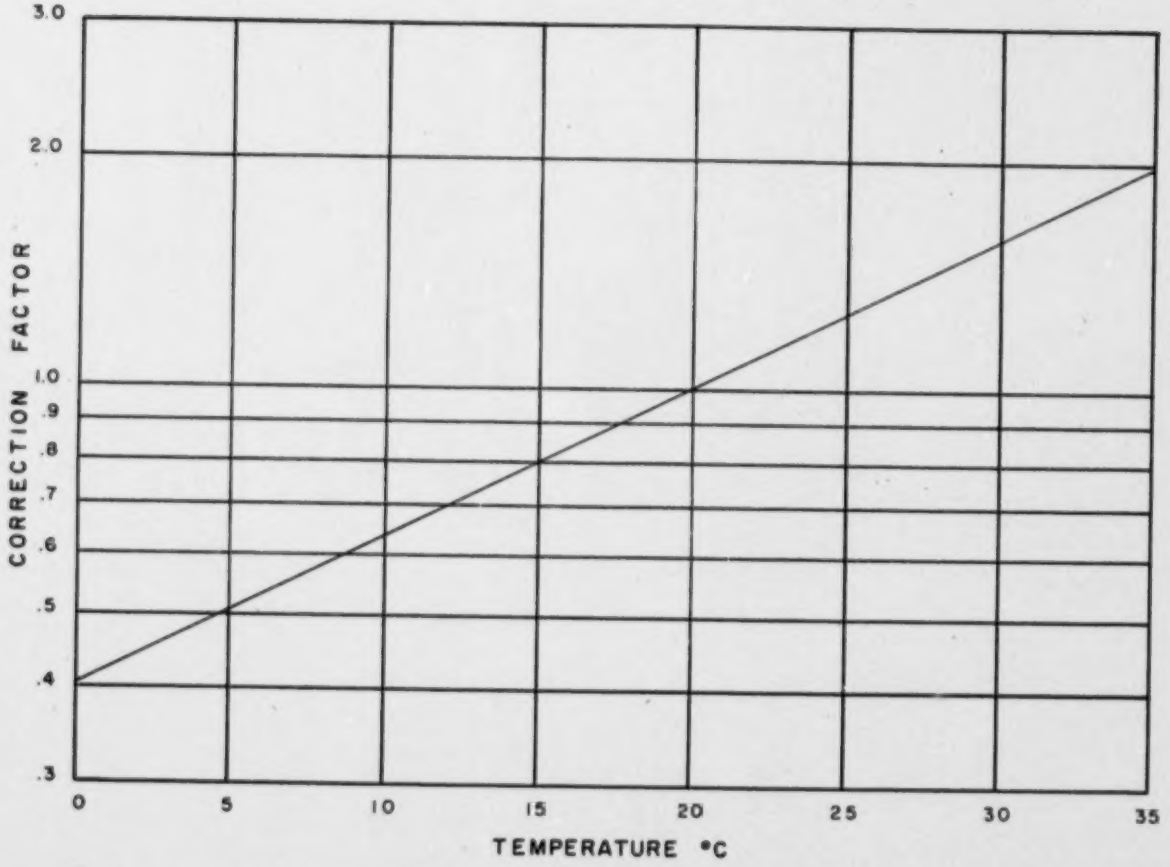
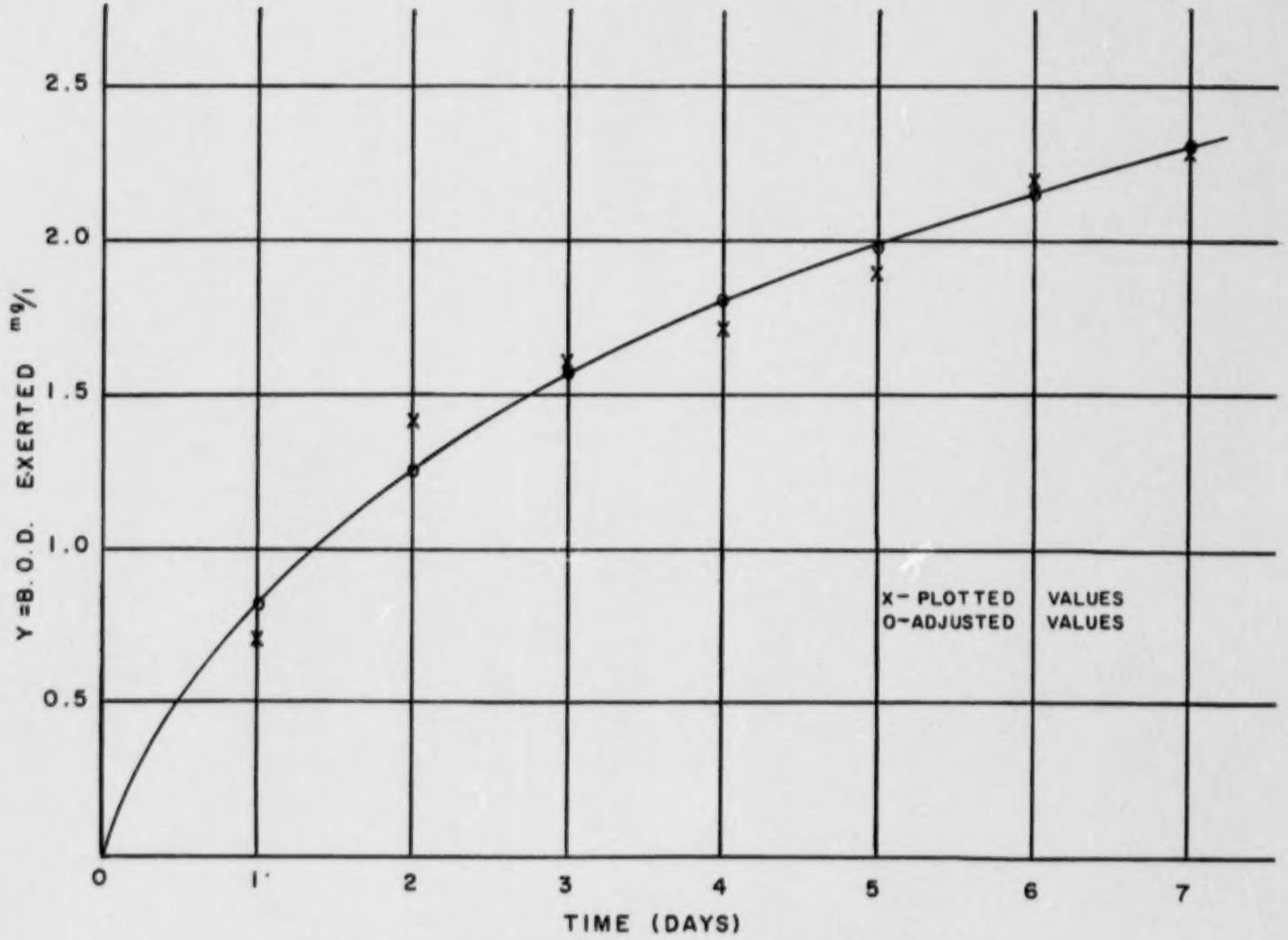


Figure 4

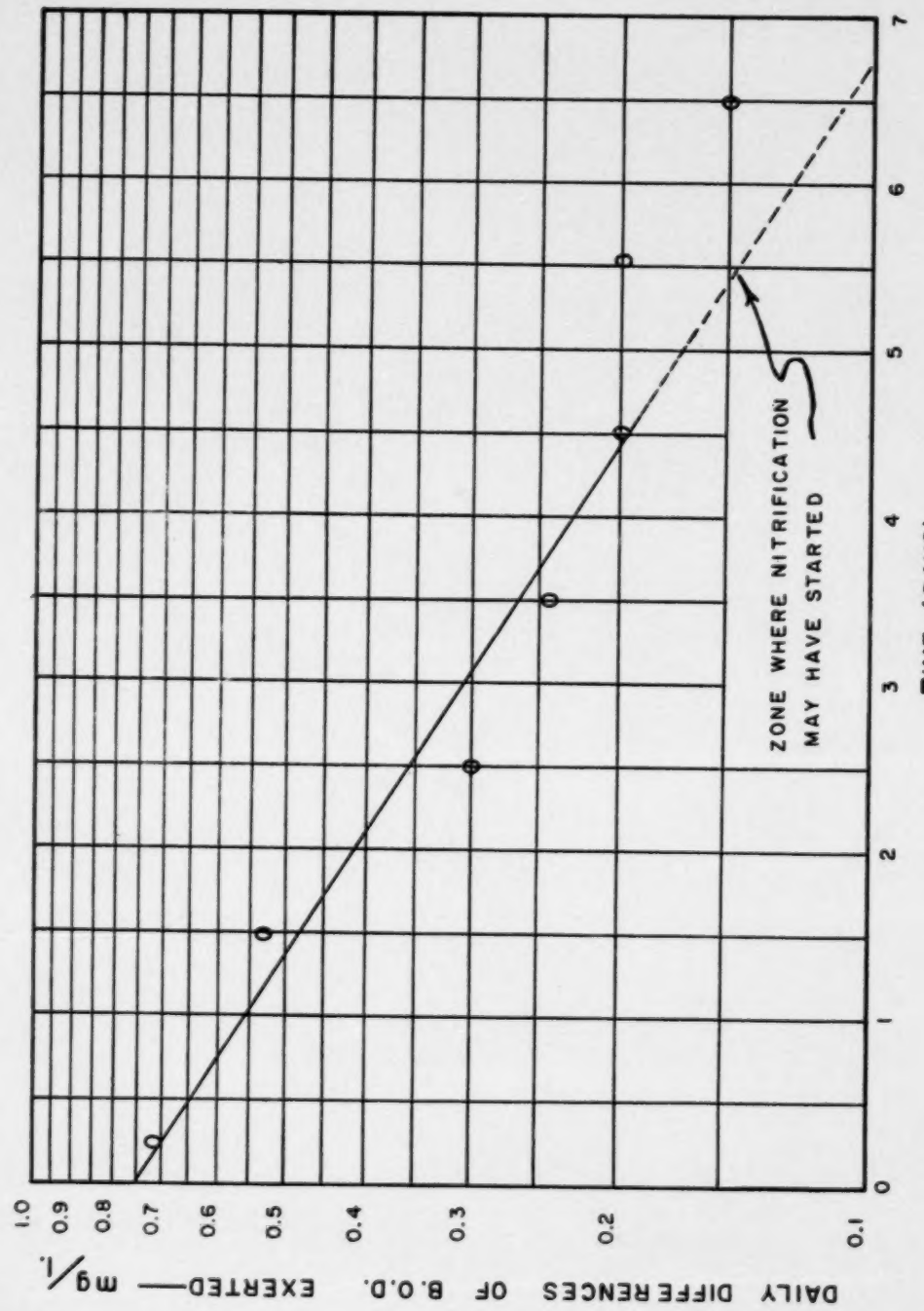
TEMPERATURE CORRECTION FACTOR FOR K_1 or k_2

$K_T = K$ at 20° C x 1.047 (T-20)



DAILY DIFFERENCE DETERMINATION
B.O.D. EXERTED vs TIME

FIGURE 5a

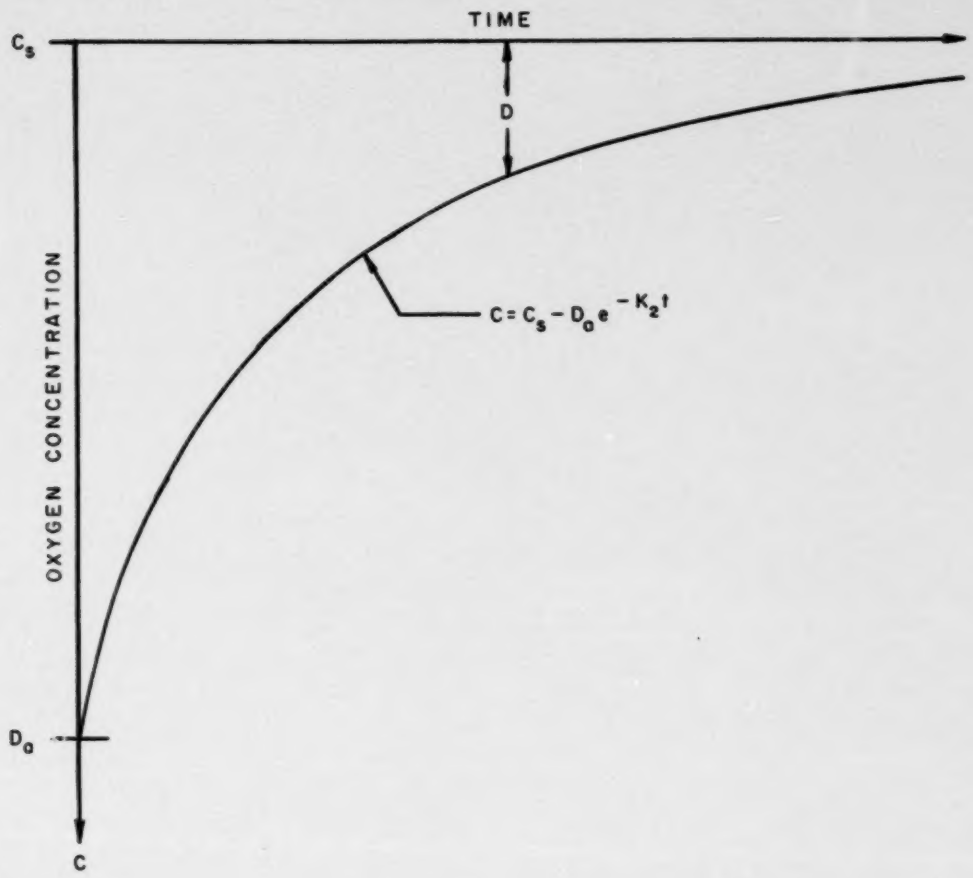


DAILY DIFFERENCE DETERMINATION
B.O.D. DIFFERENCE vs TIME

FIGURE 5b

OXYGEN BALANCE PROFILE
STREAM DISTANCE IN MILES
POUNDS of B.O.D./DAY
POINT OF WASTE DISCHARGE
POINT OF WASTE DISCHARGE
POINT OF WASTE DISCHARGE
①
②
③
A
B'
C'
D
E
F'
G'
H
I
J'
K'
L
M

Figure 6



C_s = saturation D.O. concentration at existing temperature and pressure

C = observed D.O. concentration

D = D.O. deficit at any time t, $C_s = C$

D_a = initial D.O. deficit

K_2 = coefficient of reaeration (sorption of O_2 into water) base "e"

t = time lapse

GRAPH SHOWING STREAM REAERATION REACTION

Figure 7

EXAMPLE:

K_2 (at any T) = K_2 (at 20°) x (FACTOR FROM GRAPH

K_2 (at 15°) = K_2 (at 20°) x (0.92)

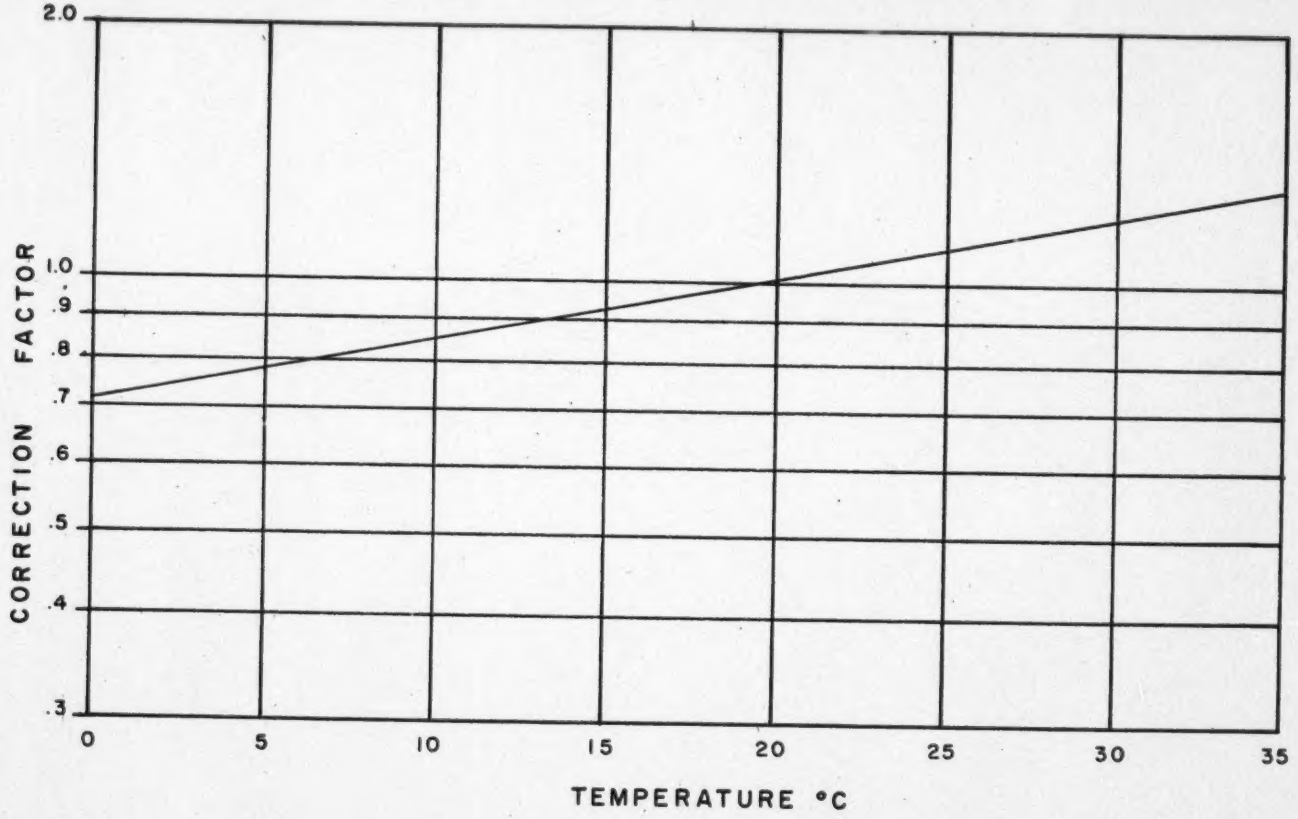
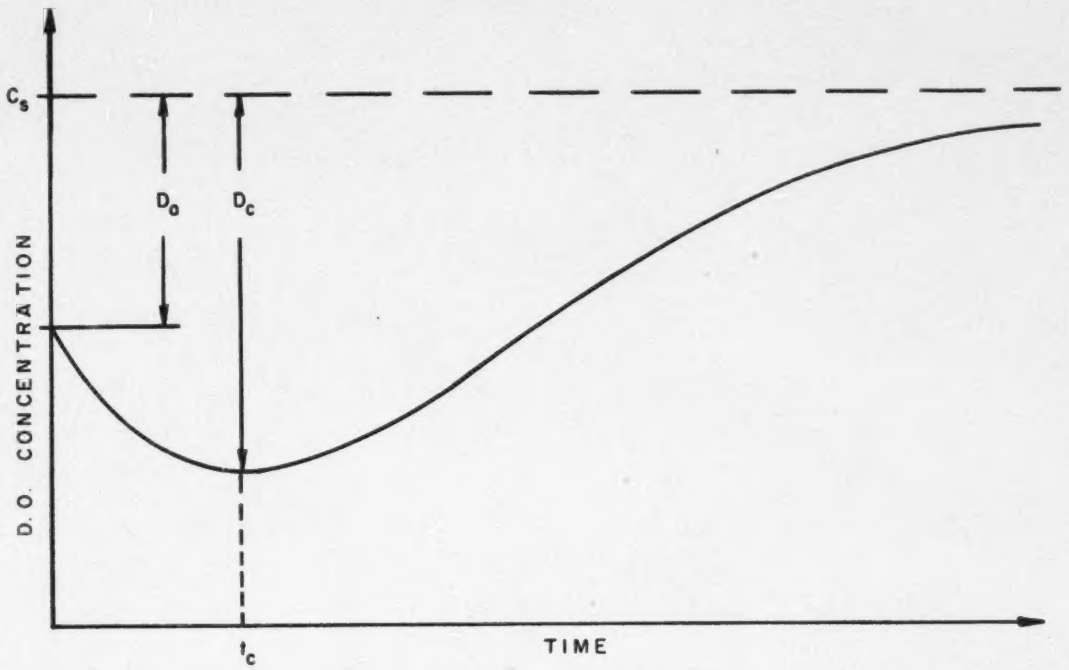


Figure 8

TEMPERATURE CORRECTION FACTOR FOR K_2 or k_2

$K_2 = K_2$ at 20° C x 1.017 (T-20)



C_s = saturation D. O.
D_a = initial D. O. deficit
D_c = critical deficit = minimum value D. O.
t_c = time required to reach minimum D. O.

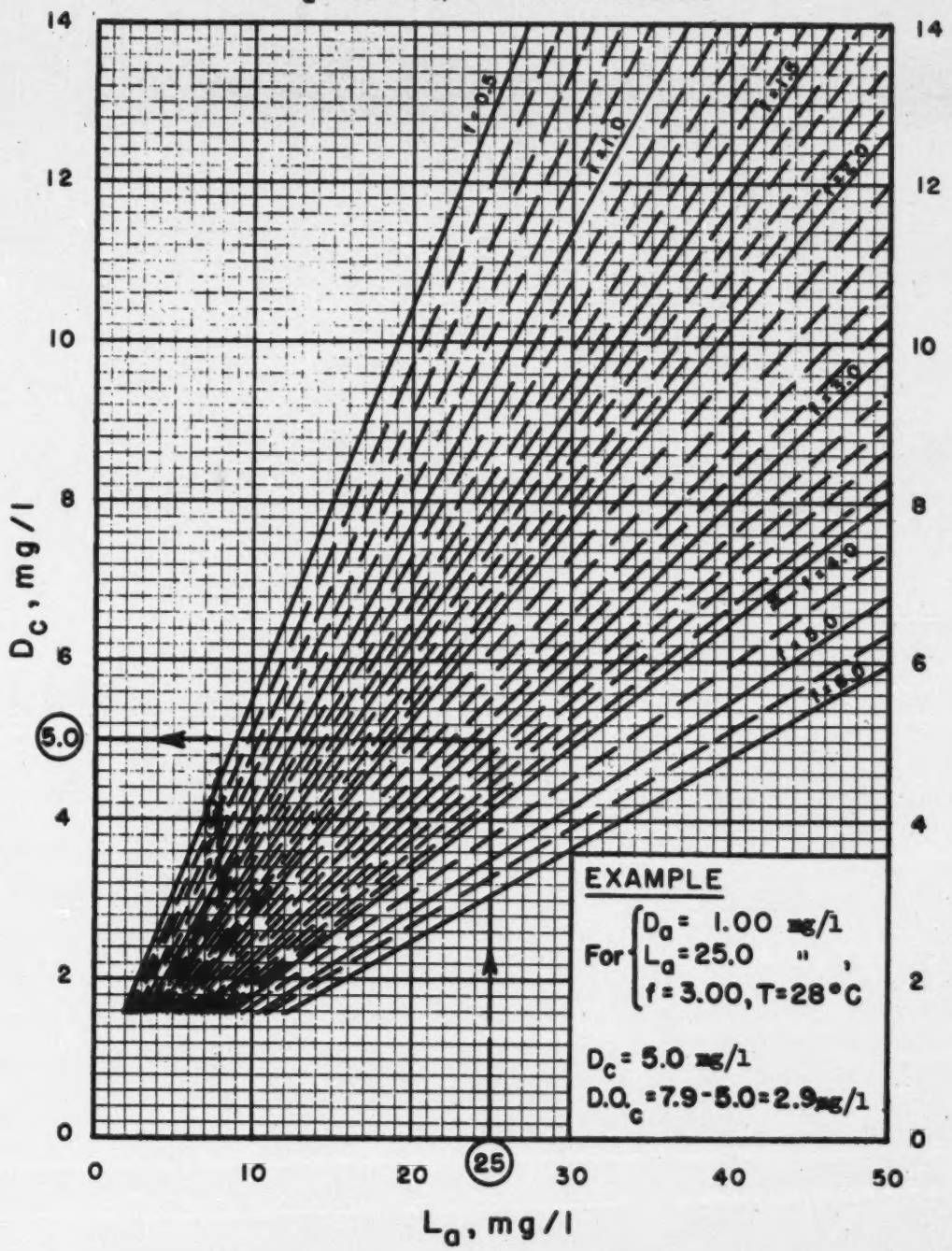
GRAPH OF A TYPICAL OXYGEN SAG REACTION

Figure 9

$$D_c = \frac{L_a}{f} \cdot 10^{-k_1 t_c}$$

$f = k_2/k_1$

D_a = 1.00 mg/l = initial deficit



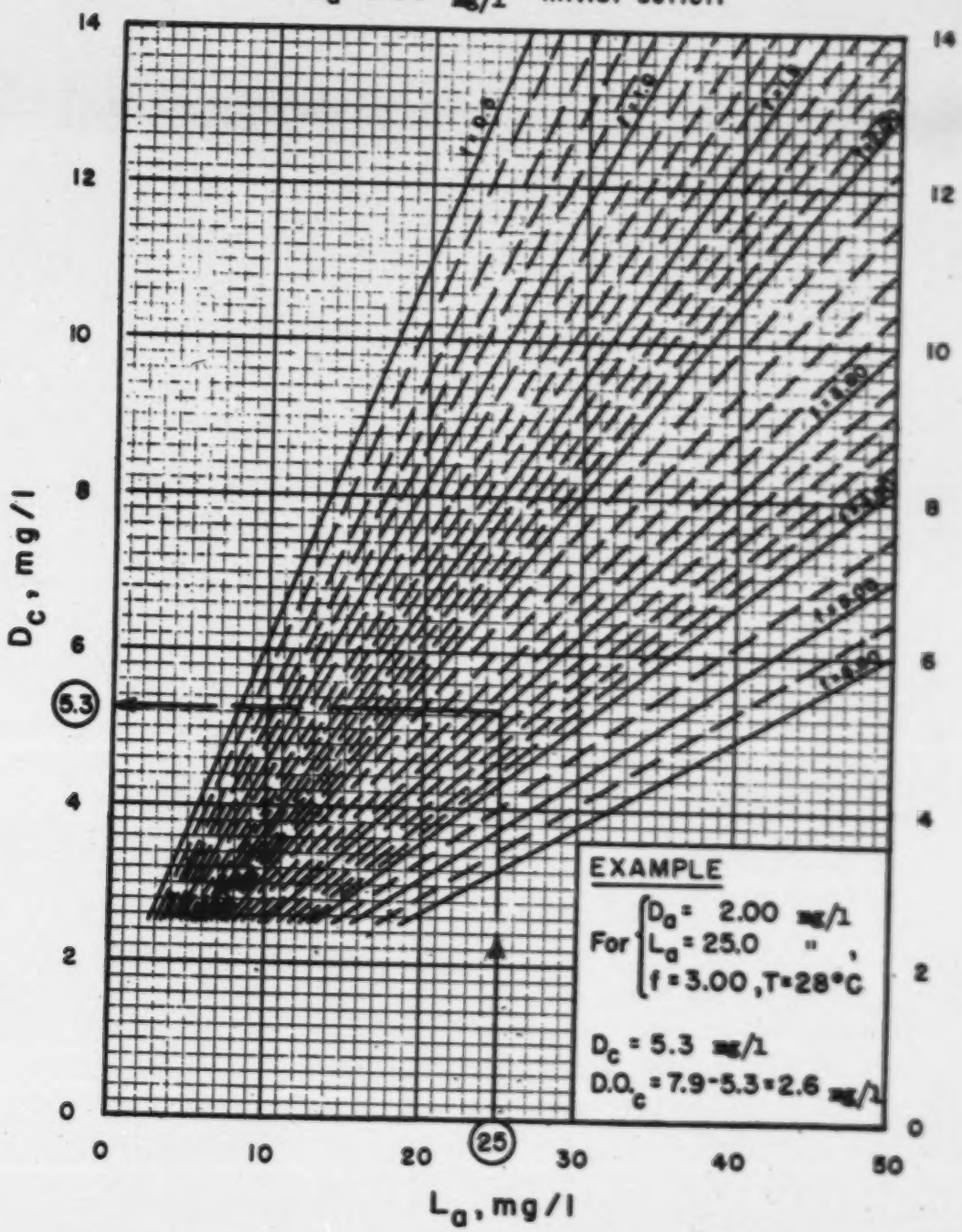
CRITICAL DEFICIT, D_c vs POLLUTION LOAD, L_a

Figure 10a

$$D_c = \frac{L_a}{f} \cdot 10^{-k_1 t_c}$$

$f = k_2/k_1$

$D_a = 2.00$ mg/l = initial deficit



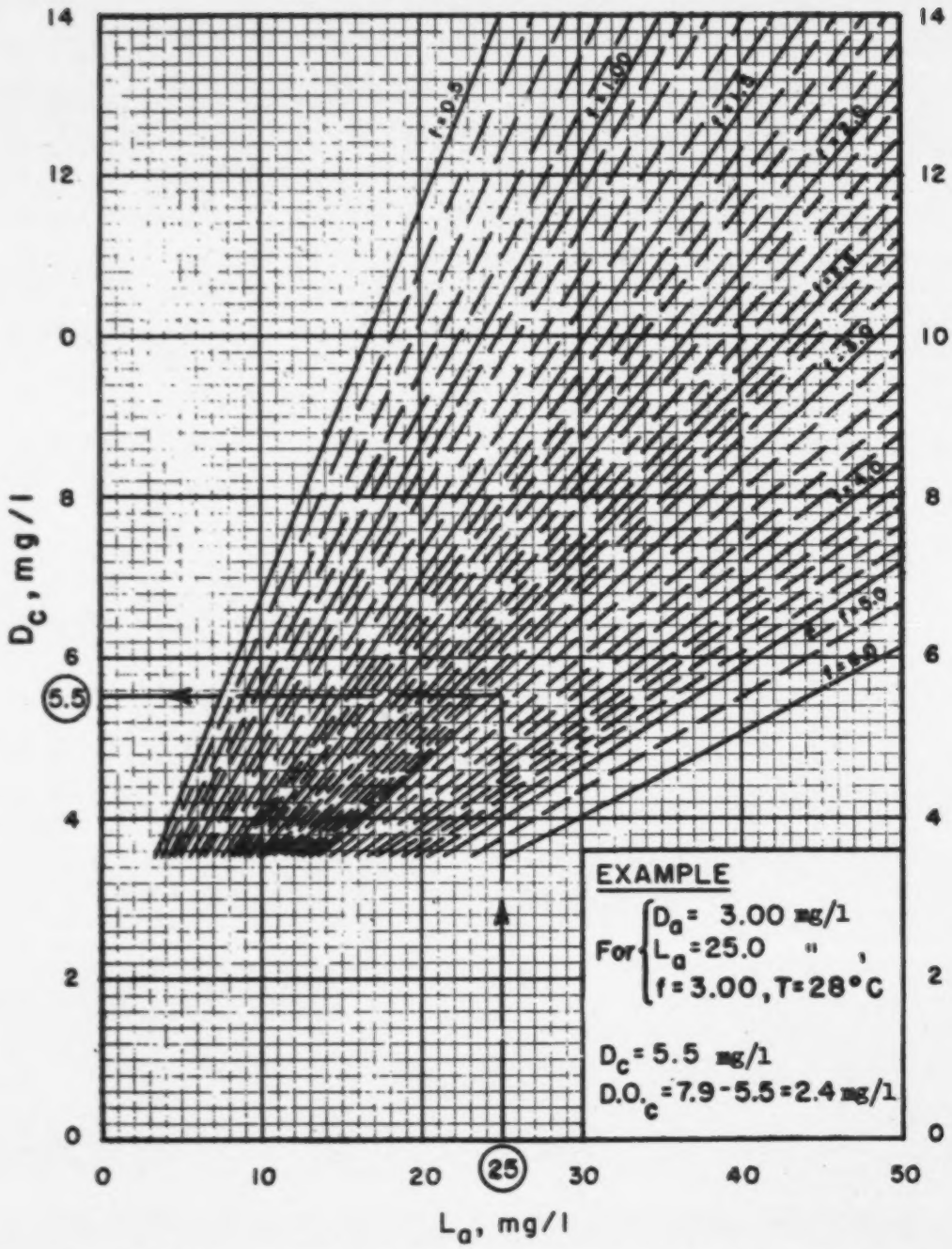
CRITICAL DEFICIT, D_c vs POLLUTION LOAD, L_a

Figure 10b

$$D = \frac{L_a}{f} \cdot 10^{-k_1 t_c}$$

$f = k_2/k_1$

$D_a = 3.00$ mg/l = initial deficit



CRITICAL DEFICIT, D_c vs POLLUTION LOAD, L_a

Figure 10c

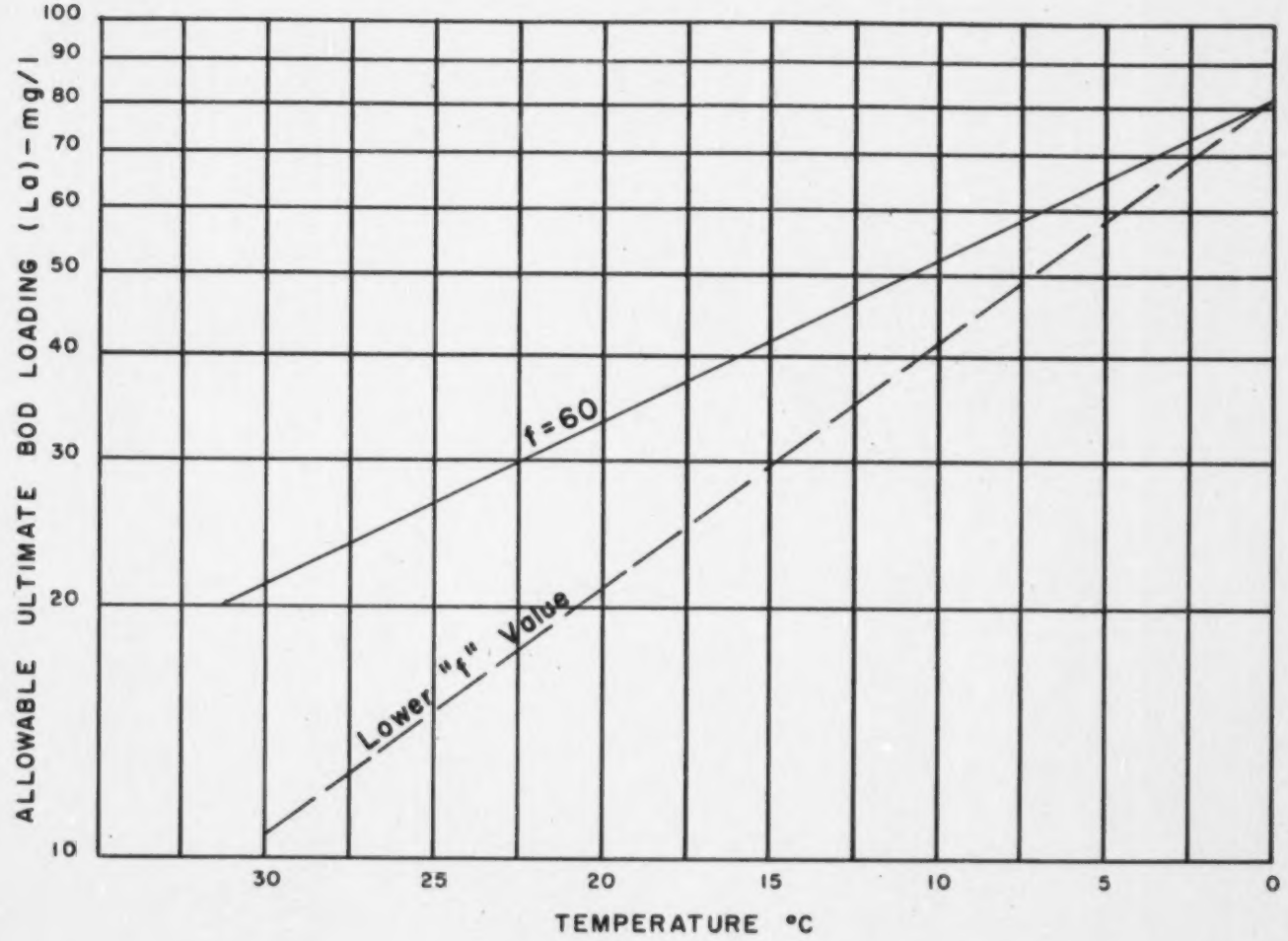
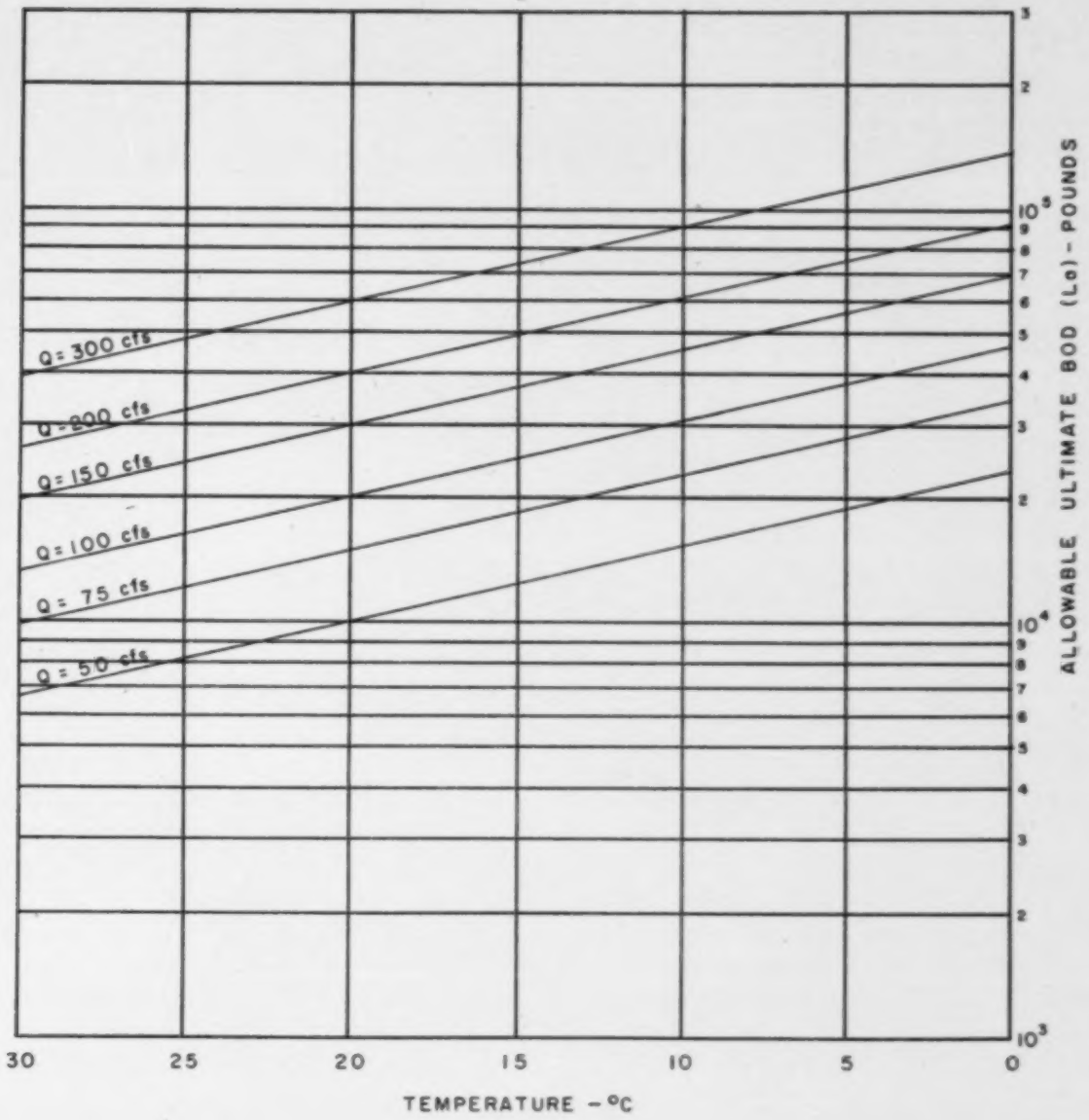


Figure 11

LOG BOD LOADING
VS
TEMPERATURE °C



ALLOWABLE BOD
LOADING vs TEMPERATURE & FLOW

Figure 12